

Exercise 9 – 18.12.2025

Time-dependent behavior of geomaterials – Visco-elasto-plastic stress-strain constitutive models

Problem 1 – Maxwell's model

1. Derive the differential equation, in terms of stress and strain, governing a creep or relaxation problem according to Maxwell's model, analytically.
2. A stress σ_0 is applied instantly $t_0=0$ and kept constant over time on a clay, use a Maxwell model to describe a creep process. Answer the following questions:
 - a) Based on the equation obtained in question 1, provide the relationship describing the evolution of strain over time.
 - b) Consider the loading condition reported in Figure 1 (total stress equal to 0.1 MPa is applied instantly at $t=0$, it is kept constant and is removed instantly at $t=72$ h) and the following values of parameters: Young's modulus: 15 GPa, viscosity coefficient: 10^4 MPa · h ($3.6 \cdot 10^{13}$ Pa · s).

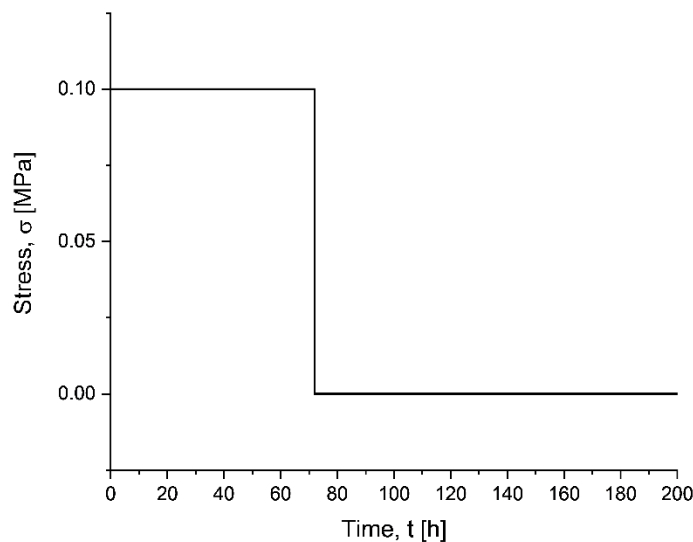


Figure 1. Loading condition.

Determine:

- b1) the instantaneous strain at $t=0$ h;
 - b2) the total strain at $t=24, 48$ and $72, 96$ h;
 - b3) Comment on the evolution of the total strain over time both before and after 72 h.
3. In case the stress is not removed at $t=72$ h, what would be the strain predicted by the model when $t \rightarrow \infty$? Is this realistic?

Problem 2 – Kelvin-Voigt model

4. Derive the differential equation, in terms of stress and strain, governing a creep or relaxation problem according to the Kelvin-Voigt model, analytically.

5. A stress σ_0 is applied instantly $t_0=0$, and kept constant over time on granite, use a Kelvin-Voigt model to describe a creep process. Answer the following questions:
 - a. Based on the equation obtained in question 4, provide the relationship describing the evolution of strain over time.
 - b. Consider the loading condition reported in Figure 1 and the following parameters: Young's modulus: 20 GPa, viscosity coefficient: 10^{19} Pa · s.

Determine:

- b1) the instantaneous strain at $t=0$ h;
 - b2) the total strain at $t=24, 48$ and $72, 96$ h;
 - b3) Comment on the evolution of the total strain over time both before and after 72 h.
- c. Repeat the computations of question 5b for the clay considered in Problem 1 (Young's modulus: 15 GPa, viscosity coefficient: 10^4 MPa · h). Make a comparison with the predictions obtained according to the Maxwell's model.